5-DOF harmonic frequency control using contactless magnetic guides

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ABSTRACT

In order to attenuate harmonic disturbances of a milling process, the contactless magnetic guide of a milling machine prototype is being used as a sensor and actuator at the same time. This paper gives an outline of an algorithm that detects such harmonic disturbances through the guide's inherent sensory capability and calculates a corresponding correction force in five degrees of freedom while adapting to the disturbance in phase, amplitude and frequency. The computed compensation force is transmitted to the spindle using the magnetic guide. The algorithm is designed with regard to low computational cost and examined in simulation and by experiment. The resulting surface is also analyzed.

1. Introduction

As a matter of principle, milling machines are subject to a process-induced vibration excitation during operation. On the basis of a full-size three-axis milling machine prototype, this paper demonstrates the active attenuation of excitations of the spindle slide originating in a milling process. It will be shown that by attenuation of the spindle slide the resulting workpiece's surface can be improved. The milling machine features a contactless magnetic guide, which is being employed as a sensor and actuator at the same time to reduce vibrations. Subsequent to an overview of the nature of the disturbances in a milling process, a short survey on mechatronic systems in machine tools is given. Then the test rig is explained and the algorithm and its application onto the machine tool are discussed in the following. The results are analyzed by means of measurements using simulation and experiment. This work closes with a short summary and outlook.

1.1. Disturbances in milling processes

The milling process uses a rotating tool and is therefore of a periodic nature. The impact impulses of the cutting edges striking the material lead to a periodic forced excitation of the tool, the workpiece and the whole machine structure. In case of equal tooth spacing, this tooth passing frequency will amount to a multiple (according to the number of teeth) of the rotational frequency. Unequal tooth spacing can distribute the energy throughout a wider range of frequencies as investigated by Doolean [1]. Presuming run out, the rotational frequency will be clearly visible within the spectrum of a milling process. Imperfect roundness or differing wear of the cutting edges will also produce an excitation proportional to the rotational frequency. Mainly originating in non-sinusoidal excitation, higher harmonics of both, the rotational and the tooth passing frequency are also visible in the process forces, but usually in lower amplitudes. Since the impulse-shaped cutting forces form a wide-band stimulus, eigenfrequencies of the cutter, the workpiece and the machine tool may also appear. As long as the process remains stable, the rotational and the tooth passing frequency constitute the dominant excitation. In case of unstable processes such as regenerative chatter, the eigenfrequencies of the tool and the machine become predominant for the overall system dynamics and influence the chatter frequency. An essay on different chatter frequencies in milling processes can for example be found in [2] by Insperger et al. (2003).

Within this article, the investigated processes remain stable. Nevertheless, the described disturbances do not only periodically excite the cutting tool but also the underlying machine frame. Depending on its stiffness, the structure will answer with an oscillation, possibly leading to an amplified disturbance at the workpiece compared to an ideally stiff machine. All of these disturbances may be visible in the resulting surface of the workpiece.

1.2. Mechatronics in machine tools

The extension of a machine tool by additional sensory or actuator capabilities has been gaining impetus as promising means to raise the productivity and stability of manufacturing processes. A comprehensive overview of mechatronic systems in machine tools is given by Neugebauer et al. [3] and presents a variety of possible applications. Regarding milling machines,
general interest is on the one hand directed towards detecting the process forces to perform process and machine surveillance. On the other hand, beneficially influencing the machine’s vibration characteristics is highly desirable, since these characteristics limit the achievable operational speed and precision to a great extent.

Current approaches to introduce such new subsystems into machine tools may thereby be distinguished into solely sensory solutions and active or passive combined actor/sensor systems. The knowledge of the process forces permits detailed process diagnosis, thus among the first group this has been subject to extensive investigations. One possible solution is the integration of displacement sensors (e.g. eddy current or capacitive) into the spindle as shown by Albrecht et al. [4]. Another approach is monitoring the spindle and motor currents and combining it with a friction model as shown by Lapp [5]. Such systems have become popular in the industrial application as they can be integrated into the NC-control with a limited effort and allow simple process surveillance such as missing tool detection or even material changes and tool wear. Due to presumably imprecise wear models and relatively slow sampling intervals, today’s commercially available solutions still suffer from low precision and resolution in time and amplitude compared to additional sensory equipment. This issue can be addressed through a combined process simulation as presented in [6] by Denkena and Schmidt (2006), which can be parameterized according to the instantaneous operational data from the NC-control. An attachment of rotating spindle-bound wireless dynamometers leads to precise torque and force measurements, but it comes with some significant disadvantages concerning cost, stiffness and handling. It is therefore restrained to laboratory usage. Sensors integrated into the workpiece holder or clamping system are more likely to ease this issue. Litwinski et al. use such a clamping system for diagnostic purposes in [7].

Beyond sole measurements, an adjustable alteration of the machine’s dynamic behavior requires some kind of an actuator. The electric feed drives of a machine tool do usually offer relatively low dynamics (usually <50 Hz at ~3 dB), so additional highly dynamic actuators are being installed in machine tools. Piezo-electric devices and low-inductive electromagnetic actuators in particular have proven to be well-suited for this task. Again both, the spindle and the workpiece can be targeted. For example, a piezoelectric holder for the workpiece presented in [8] by Rashid and Niculescu (2006) allows quick workpiece relocation in two directions. Here, the dynamic properties do rely on the mass of the workpiece. In [9] Denkena et al. (2006) relocate the whole main spindle in three directions with the help of piezoelectric actuators, thus the dynamic properties are decoupled from the workpiece’s weight. Primarily targeting non-rotating tools like those being used in turning applications, highly dynamic actuation systems have been referred to as Fast Tool Servos (FTS). Such a tool that is actuated by electromagnets has been constructed by Trumper [10]. Brecher and Schauerte (2008) have presented a piezoelectrically actuated boring bar in [11], where the blades are individually controlled and vibration damping can be achieved. Rotational electromagnetic bearings have been successfully used for vibration detection and process influence, such as by Kern et al. [12].

2. Test rig

A new machine design approach which for the first time integrates an adaptronic linear electromagnetic guide as a core component into a full-size three-axis milling machine has been developed at IFW in collaboration with IDS and is presented in [13]. An image of the machine is shown in Fig. 1. In this machine concept the spindle is installed in a Z-axis slide (see Fig. 2) that is guided employing the principle of electromagnetic levitation. The X- and Y-axis are guided by conventional linear ball bearings. As an additional feature, each machine axis is driven by linear direct drives, allowing very high axis accelerations between 4.3 and 4.7 g. The stabilization of the Z-slide in five degrees of freedom (DOF) represents the mechanical guidance function and is achieved by a state-space controller using Kalman filtering. To control the magnetic guide, the air gaps at each electromagnet are measured and transformed into a generalized coordinate system \((q_1, \ldots, q_5)\) representing five of the slide’s six physical rigid body DOF as shown in Fig. 2. Additionally, accelerations in these five DOF are being measured by multi-axis acceleration sensors on the front and back sides of the slide.

Based on the instantaneous air gaps and accelerations, a normalized position controller calculates corrective forces in each DOF. These are then inversely transformed back into forces for the individual electromagnets and applied to the guide. The controller is currently running at 5 kHz and implemented on a Power-PC platform running at 366 MHz. In its traverse direction the Z-axis’ linear direct drives are operated by a standard Siemens 840D industry controller. In result, each DOF of the spindle is being actively controlled. There is no physical contact between the spindle and the machine frame, due to the contactless guide and drive. It is therefore possible to relocate the whole spindle by dynamic adjustment of the air gaps, as it has been shown by Kallage [14]. This can be used to significantly improve machine precision. As a matter of principle the magnetic guide takes direct part in the distribution of forces. Thus, disturbances and forces at the tool center point (TCP) may be detected or impressed through the active guide.
3. Control of harmonic frequencies

With the main disturbance being strictly periodic, it is advantageous to apply an algorithm that is inherently periodic itself. The spectrum of a milling process as described above consists of several harmonic oscillations, each of which is characterized by frequency, amplitude, and phase. Supposing a single harmonic disturbance, a correction signal with fitting amplitude, frequency, and a phase shift of 180° can in theory fully compensate the excitation signal. With knowledge of the respective transfer function it becomes possible to excite the system in a way that the disturbance visible at the system's output is minimized. As a result, the targeted harmonic disturbance is attenuated. Such harmonic damping has for example been shown by Brecher et al. [15]. Following the idea of repetitive control as described by Bristow et al. [16], the source signal can be adjusted after every completed cycle (e.g., an oscillation). Based on the estimation of the rotational speed, a controller as suggested by Lee et al. [17] may be used for this purpose. Ruskowski (2004) has applied a harmonic damping approach to a laboratory linear magnetic guide in [18] and demonstrated such damping in one DOF.

The algorithm that is presented in this paper does not rely on a given rotational speed and firstly detects harmonic disturbances of the spindle by analyzing the displacement signal of the magnetic guide. It then calculates a correctional force that is transmitted to the spindle slide using the guide's electromagnets. With the goal of limited computational cost the disturbance identification is performed in just one DOF, but nevertheless a compensation force is separately computed and applied to each of the guide's five DOF. The compensation force is permanently adapted to the respective disturbance in amplitude, phase and frequency during operation.

Following an explanation of the algorithm, its behavior is firstly shown by means of a simulation. Afterwards, experimental results from the implementation on the actual machine are being looked at in more detail. Finally, first examinations on effects of the 5-DOF harmonic compensation to the workpiece's surface are discussed.

3.1. Algorithmic framework

A structure diagram of the 5-DOF harmonic compensation is shown in Fig. 3. The algorithm has been applied to the aforementioned full-size milling machine with magnetic guides. After an FFT-based initial identification of the disturbance in a preselected degree of freedom $i$, the major disturbance frequencies are known and sorted by their amplitudes. Then these amplitudes are compared to a boundary value and in case of exceeding the threshold an oscillator is activated. The number of oscillators is solely limited by the available computing power and memory. Each oscillator immediately starts adjusting to the disturbance frequency in one DOF $n$ while adapting to phase and amplitude is performed in all five DOF.

In Fig. 3 the different oscillators are represented by the blocks ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The compensation signal has to be calculated synchronously to the different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M’. In order to minimize the calculation time, different step times are used for the different calculations. The ‘osc 1’ to ‘osc M'. Hence, the position measurement is colored by the frequency response function (FRF) of the mechanic structure of the spindle slide. Fig. 4 shows a comparison between the true compliance at the TCP in the horizontal $X$-axis of the machine and the compliance seen at the slide's position sensors in the transulatory horizontal DOF $q_1$. Apart from the generally higher damping seen in $q_1$, the structure possesses a dominant eigenfrequency at approx. 251 Hz. This has been identified as the first eigenfrequency of the spindle holder (depicted in Fig. 2) and it reaches the air gap sensors with a particular high damping.

Thus, experiments have so far been restricted to the frequency range below this eigenfrequency. The complete integration of the slide's structural dynamics into the controller is currently being researched and will widen the available frequency range in the future.

3.3. Disturbance identification

The harmonic control relies on the presumption that the disturbance $d(t)$ at the input of the system can be described by a not taken at the TCP directly, but from the air gap sensors of the spindle slide. The input data vector to the algorithm $y(t)$ is therefore taken from the closed-loop controller and consists of the slide’s excitation in the five DOF $q_1, \ldots, q_5$. Hence, the position measurement is colored by the frequency response function (FRF) of the mechanic structure of the spindle slide. Fig. 4 shows a comparison between the true compliance at the TCP in the horizontal $X$-axis of the machine and the compliance seen at the slide’s position sensors in the transulatory horizontal DOF $q_1$. Apart from the generally higher damping seen in $q_1$, the structure possesses a dominant eigenfrequency at approx. 251 Hz. This has been identified as the first eigenfrequency of the spindle holder (depicted in Fig. 2) and it reaches the air gap sensors with a particular high damping.

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discrete sum of spectral lines:

\[ d(t) = \sum_{k=1}^{N} d_k(t) = \sum_{k=1}^{N} d_k \cos(\omega_k t + \varphi_k) \]  

(1)

With complex amplitudes this equation can be rewritten as

\[ d_k(t) = \frac{1}{2} \left[ D_k e^{j\omega_k t} + D_k^* e^{-j\omega_k t} \right] \]  

(2)

The compensation signal may be described in the same way as

\[ r_k(t) = \frac{1}{2} \left[ R_k e^{j\omega_k t} + R_k^* e^{-j\omega_k t} \right] \]  

(3)

The signal that can be measured at the output of a system with the discrete transfer function \( G(j\omega_k) \) becomes

\[ y_k(t) = \frac{1}{2} G(j\omega_k) [D_k + R_k] e^{j\omega_k t} + \frac{1}{2} G^*(j\omega_k) [D_k^* + R_k^*] e^{-j\omega_k t} \]  

(4)

Compensation is accomplished when \( R_k = -D_k \) and amplitude and frequency correspond.

As a prerequisite the disturbing frequencies have to be identified. This is accomplished, e.g. by a Discrete Fourier Transformation (DFT). By calculating the absolute values of the identified frequencies one can detect the disturbances arising from a milling process and an oscillation of the complex amplitude

\[ Y_k(t) = Y_k e^{j\Delta \omega_0 t} \]  

(10)

The difference in phase can be determined by two consecutive identifications of the amplitude:

\[ \Delta \varphi_k = \Delta \omega_0 T_{oz} \]  

(11)

With this information it is possible to adapt the frequency with the use of the following equation

\[ \tilde{\omega}_k[n + 1] = \tilde{\omega}_k[n] + \mu \Delta \varphi_k \]  

(12)

### 3.4. Adaptation of amplitudes

For adaptation of the compensation amplitudes \( R_k \) in all five DOF an iteration scheme is used:

\[ R_k[n + 1] = (1 - \mu) R_k[n] - \mu \cdot D_k[n] \]  

(6)

The convergence of Eq. (6) is dependent on the coefficient \( \mu \).

The numerical stability of this adaptation depends on the quality of the applied system model. If an error of the Amplitude \( \Delta G \) and an error of the phase \( \Delta \varphi \) are presumed the stability condition amounts to

\[ 0 < \mu < 2\Delta G \cos(\Delta \varphi) \]  

(7)

To get the amplitude of a determined frequency \( \omega_k \) in the time domain a demodulation is used. Therefore the signal \( y(t) \) in the time domain is multiplied with \( e^{-j\omega_k t} \) and gives the modulated signal

\[ \lambda(t) = y(t) e^{-j\omega_k t} = \sum_{i=1}^{N} \frac{1}{2} \left[ Y_i e^{j(\omega_i - \omega_k) t} + Y_i^* e^{-j(\omega_i + \omega_k) t} \right] \]  

(8)

Finally, the demodulation by a low pass filter with the frequency \( \omega_0 = \omega_k \) delivers the desired amplitude

\[ Y_k = \overline{\lambda}(t) \]  

(9)

### 3.5. Adaptation of frequencies

Because of the limited accuracy of the DFT and due to possibly changing disturbance frequencies during the process an adaptation of the frequency of the correction signal becomes necessary. Supposing an error in the frequency of \( \Delta \omega \) this will lead to an oscillation of the complex amplitude

\[ Y_k(t) = Y_k e^{j\Delta \omega_0 t} \]  

(10)

### 3.6. Output

The output vector \( \tau(t) \) is the compensation force in the five DOF \( q_1, \ldots, q_5 \) that shall be applied to the spindle slide in order to suppress the excitation. It is in the same sense applied through the guide’s eight electromagnetic actuators. Therefore, the (inverted) disturbance FRF for each DOF has to be determined by a priori measurement. Fig. 5 shows such a FRF for the horizontal X-axis of the machine, both at the TCP and at the air gap sensors. Similar to the input side, the slide’s eigenfrequency at 251 Hz does not appear in the ‘internal’ sensor measurement.

The desired compensation force of the respective electromagnetic is finally added to the force calculated by the position controller that is necessary to stabilize the slide.

### 4. Simulation

The described procedure shall firstly be demonstrated by means of simulation. Therefore, a simulation environment in Matlab/Simulink has been set up. The behavior of the magnetic guide is modeled in Simulink, while the compensation algorithm is implemented in a Matlab m-function.

In the following example two harmonic disturbances with 50 Hz and 90 Hz are applied to all DOF, each with a different amplitude. The simulated control frequency of the magnetic guide is 5 kHz and the FFT is taken over 2048 samples, which results in a duration of approximately 0.4 s. After this time the first oscillator with 90 Hz is started. After 0.8 s the second oscillator with 50 Hz is started. The amplitudes of the five DOF are shown in Fig. 6. It can be seen that the disturbance is nearly compensated after 1.5 s and it is completely leveled out after 3 s.

![Fig. 5. Disturbance FRF at TCP and in q1.](image-url)
The frequency adaptation process of the two activated oscillators is illustrated in Fig. 7. The first approximation amounts to 49 Hz and within several steps it gradually improves to the applied 50 Hz.

Fig. 8 shows the amplitudes of the compensation signal of the second oscillator in the first and second degree of freedom. After showing an initial overcompensation both oscillators recede to the actual disturbance amplitude, which is finally reached after approximately 1 s.

In conclusion, simulation has shown the algorithm is working and able to compensate disturbances in each DOF.

5. Experiment

In order to validate the simulative results and to further investigate practical effects, the harmonic controller has been implemented on the aforementioned milling machine with magnetic guide. Three different types of examinations have been carried out:

- Internal sinusoidal excitation through the magnetic guide by application of a disturbance signal within the control loop.
- External sinusoidal excitation by a mechanical shaker.
- Milling tests.

Since the operational code has not yet been fully optimized for execution speed, the currently available computing power confines the number of oscillators to one. This is however sufficient to show the functionality of the harmonic control in five DOF.

5.1. Harmonic excitation

At perfectly harmonic excitation applied internally or externally by means of a shaker, the developed harmonic control of the magnetic guide shows very good damping ratios after adaptation as can be seen in Fig. 9.

On the left side there is a plot of the measured disturbance amplitude in each DOF, and on the right side the applied compensation force is displayed. The external disturbance is switched on at 0.4 s and switched off at 6.4 s. Both the translational \( q_1 \) and \( q_2 \) and the rotational \( q_3, q_4, \) and \( q_5 \) DOF were excited by the shaker and could successfully be attenuated.

The result may be further explained using a closer look on the pitch axis \( q_4 \) of the Z-slide as given in Fig. 10. Following an initial phase of frequency identification the compensation force is applied to the slide, which results in a noticeable reduction of the disturbance amplitude. After switching off the external excitation, the compensation force causes an excitation of the spindle itself, which is visible in the disturbance amplitude. The algorithm now

![Fig. 6. Simulated amplitudes of the DOF \( q_1, \ldots, q_5 \).](image1)

![Fig. 7. Frequency adaptation for two oscillators.](image2)

![Fig. 8. Amplitude adaptation of oscillator 2 in the DOF \( q_1 \) and \( q_2 \).](image3)

![Fig. 9. Amplitudes of all controlled DOF during shaker excitation at 60 Hz.](image4)
detects the missing external disturbance and shuts down the applied force.

5.2. Validation by milling tests

Apart from ideal harmonic excitation, milling tests have been carried out with the aim of examining the behavior of the algorithm in the case of a more complex disturbance signal. At first, a groove milling process according to Fig. 11 has been selected to test the algorithm.

As a predominant disturbance, the tooth passing frequency has been automatically targeted by the compensation algorithm. Not surprisingly, this disturbance frequency does not appear in every DOF with the same intensity. Thus, damping ratio varies over the DOF with the best results in the rotational DOF.

As an example the resulting amplitude and corrective force of the yaw axis are shown in Fig. 12. After the cutting tool gets in contact with the material at \( t = 3 \) s and during identification which is finished after another 0.4 s, the cutting process causes an excitation of approximately 8 \( \mu \)rad. As soon as the compensation force is applied, the disturbance amplitude is reduced by about 50% or by 4 \( \mu \)rad through harmonic control. After the tool leaves the material, the compensation force is shut down quickly.

The effect of the compensation may be further illustrated with a close-up view on two spindle revolutions as given in Fig. 13. The upper half shows the deviation of the spindle slide in DOF \( q_1 \), and the lower half the applied compensation force. On the left side no compensation force is applied, resulting in a close-to-sinusoidal shaped distortion arising from the tooth passing. With the compensation force applied as shown on the right side, the distortion amplitude is significantly lowered.

The effectiveness of the implemented compensation differs over the frequency range. As shown in Fig. 14, in comparison to the uncompensated slide the current implementation is beneficial, especially between 10 and 80 Hz.

It must be considered that this compensated FRF is only valid stationary for single frequencies, since currently just one oscillator is active at any point in time. Furthermore, the absolute stiffness of the system especially in this frequency range is heavily dependent on the settings of the position controller, which have therefore been held constant for all investigations.

5.3. Surface effects

A lowered amplitude of the spindle vibration could technically lead to lower artefacts on the workpiece surface. First examinations of the resulting surface have been carried out using a different milling parameter set of \( a_e = 7 \) mm, \( a_p = 2 \) mm, \( n = 3000 \) RPM, \( v_f = 320 \) mm/min. The measured roughness parameters of the face and border surfaces are shown in Table 1.

Both roughness values \( R_a \) and \( R_z \) have been reduced by up to 25% in the compensated process regarding both the face and the edge.
surfaces. Since compensation has only been applied to the five DOF of the magnetic guide and not to the Z-axis of the machine that is normal to the workpiece’s face surface, its roughness improvement is caused indirectly by the compensated pitch and yaw DOF.

To further investigate this effect, the surface has been scanned with a conoscopic laser interferometer. A resulting profile is shown in Fig. 15. It shows the face surface taken at 1.5 mm distance from the edge in the time domain. The upper part is taken from the uncompensated process and the lower part shows the compensated process.

It is visible that the surface profile has been slightly reduced in its amplitude, providing an explanation for better roughness values. Furthermore, it has changed its shape. This effect may be more comprehensively explained by a look at the frequency domain.

In Fig. 16(a) and (b) the compensation is switched off, in (c) and (d) it is switched on. Fig. 16(a) and (c) show the averaged frequency content of the surface profile with and without compensation. This has been calculated based on the known feed rate. In (b) and (d) there is a short time Fourier transform (spectrogram) given over the period of 1 s, which relates to a distance of 5.333 mm. It becomes apparent in Fig. 16(a) and (b) that the main shape of the surface is composed of oscillations of the rotational frequency, the tooth passing frequency and their higher harmonics. It can then be seen in (c) and (d) that the damping of the slide’s vibration at the tooth passing frequency has lead to reduced surface artefacts at several frequencies, although just one frequency has been damped. Corresponding to the time-domain data seen in Fig. 15, the discontinuous spectrum of the uncompensated surface has changed into a slightly more continuous spectrum, arising from a less periodically shaped surface.

6. Conclusion and outlook

In this paper an algorithm that detects and compensates harmonic disturbances through a contactless magnetic guide has been presented by means of simulation and experiment. It has been shown that through application of the developed algorithm disturbances to the machine structure such as those arising from a milling process were considerably attenuated. Only sensors and actuators that are part of the magnetic guidance system have been used in this implementation. The resulting surface has been examined with regard to the effects caused by the applied compensation force.

Since all presented practical tests have been accomplished with only one oscillator, upcoming investigations will involve simultaneous compensation of multiple frequencies. Future improvements may also result from an enhanced integration of the

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Surface roughness effects on face and border surfaces (face LM 12.5 mm, border LM 4.0 mm).</th>
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<tbody>
<tr>
<td></td>
<td>$R_a (\mu m)$</td>
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<tr>
<td><strong>Face uncomp.</strong></td>
<td></td>
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<tr>
<td>Min</td>
<td>0.366</td>
</tr>
<tr>
<td>Max</td>
<td>0.826</td>
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<tr>
<td>$\sigma$</td>
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<tr>
<td>$\varphi$</td>
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<tr>
<td><strong>Face comp.</strong></td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>0.387</td>
</tr>
<tr>
<td>Max</td>
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</tr>
<tr>
<td>Improvement $\varphi$</td>
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<tr>
<td><strong>Edge uncomp.</strong></td>
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<tr>
<td><strong>Edge comp.</strong></td>
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<td>Improvement $\varphi$</td>
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Fig. 15. Face surface profile at 1.5 mm distance from the edge, with and without compensation.

Fig. 16. Face surface profile in the frequency domain.
structural dynamics, which can further advance the results of the harmonic compensation.

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